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Original article

Real-Time Inverse Kinematics for a Two-Degree-of-Freedom Upper-Limb Rehabilitation Exoskeleton

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Abstract

Traditional numerical methods for solving inverse kinematics in rehabilitation exoskeletons often suffer from high computational latency and struggle with dynamic torque optimization during high-frequency real-time control loops.

Objective: Evaluation and testing a two-stage physics-informed feedforward neural network framework for solving the inverse kinematics of a 2-DOF upper-limb exoskeleton while simultaneously minimizing static gravitational joint torques.

Methods. An off-line multi-start optimization routine was used to generate a workspace dataset of 99,208 optimal configurations; a feedforward architecture with 135,814 parameters incorporating a custom differentiable physical gravity layer was then trained to predict joint angles and correct residual loads.

Results. Across a test set of 14,882 points, the model achieved a mean spatial positioning error of 0.385 cm (3.85 mm) with an approximate inference latency of 2.0 ms per sample, supporting closed-loop control frequencies up to 500 Hz without relying on resource-intensive online quadratic programming.

Conclusions. The proposed physics-informed learning approach effectively combines high spatial accuracy, energy-efficient torque distribution, and deterministic execution speed, providing a lightweight, low-latency control solution for wearable upper-limb rehabilitation robotics.

Keywords: rehabilitation robotics, upper-limb exoskeleton, inverse kinematics, feedforward neural network, real-time control, joint configuration, assistance trajectories.

1. Introduction

Upper-limb motor impairment constitutes a critical global healthcare challenge. According to recent estimates from the Global Burden of Disease (GBD) study [1] and the World Health Organization (WHO) Global Stroke Fact Sheet [2], stroke remains the second leading cause of death and the third leading cause of combined death and disability worldwide, with over 11.9 million new cases occurring annually and more than 93 million individuals living with post-stroke sequelae [1,2]. Motor dysfunction affects up to 75% of stroke survivors, among whom approximately 80% experience persistent upper-limb hemiparesis or total functional loss. Because upper-limb dexterity is essential for reaching, grasping, and performing basic activities of daily living (ADLs)—such as feeding, dressing, and personal hygiene—severe motor deficits drastically reduce patient independence, induce profound psychosocial distress, and impose an escalating socio-economic burden on healthcare systems.

Clinical physical therapy and neurorehabilitation rely heavily on task-specific, high-intensity, and repetitive therapeutic exercises to promote neuroplasticity and functional cortical reorganization. However, conventional manual therapy requires intensive physical exertion from therapists, leading to early clinician fatigue, variable session consistency, and severe constraints on overall clinical throughput. Wearable active-assisted robotic exoskeletons have emerged as a promising solution, offering high-frequency, reproducible, and objective therapeutic training. To ensure safe and physiologically natural interaction with an impaired human arm, the embedded control framework of an exoskeleton must continuously calculate inverse kinematics (IK) to drive the end-effector along anatomical target trajectories without exceeding safe joint limits or causing abrupt, jerk-inducing posture changes [3,4]. Foundations in kinematic modeling, differential relationships, and velocity mapping were established regarding the analytical framework for connecting operational end-effector trajectories with joint-space movements in [3,4]. Subsequently, researchers expanded these concepts to account for joint load and energy consumption during trajectory execution. Specifically, stable torque optimization was demonstrated for redundant robotic structures using short preview methods [5], while the necessity of minimizing gravity-related joint torques was highlighted to improve operational efficiency [6].

Despite significant technological advancements, integrating traditional IK controllers into lightweight, wearable upper-limb rehabilitation devices presents critical computational and biomechanical limitations [5–10]. Classical numerical IK algorithms, including Jacobian-based formulations [7], variable step-size iterative methods [8], or quadratic programming approaches, require intensive real-time computation [7,8]. These methods introduce variable execution delays, limiting their feasibility for high-frequency embedded microcontroller operation [7,8]. Furthermore, as demonstrated in [9,10], classical numerical solvers are susceptible to local minima traps and singular configuration failures near workspace boundaries, presenting potential safety risks during active patient-robot interaction.

To overcome computational latency, several investigators have explored machine learning techniques to map workspace coordinates directly to joint states. Generative methods such as IKFlow for learning diverse inverse kinematics solutions were developed in [11], while artificial neural networks demonstrated in [12,13] could successfully learn complex inverse kinematic mappings in articulated robotic mechanisms. Moreover, bionic energy-efficient neural network architectures were introduced for legged mechanisms [14], and the viability of embedding physical laws into continuous-time dynamic neural models was demonstrated in [15]. However, conventional artificial neural network architectures in the existing literature frequently treat inverse kinematics strictly as a geometric fitting problem [11–13]. By not considering static gravitational joint torques and actuator loading constraints during joint angle prediction, existing data-driven models often produce sub-optimal configurations that increase power consumption, cause motor overheating, and exert excessive gravitational resistance on the patient's supported limb. A distinct knowledge gap persists regarding the integration of differentiable physical load models directly into neural inverse kinematics architectures to optimize static gravitational torques simultaneously with spatial positioning accuracy.

To address these computational and biomechanical limitations highlighted in prior literature, this study presents a two-stage, physics-informed neural network framework for solving the inverse kinematics of a miniature two-degree-of-freedom upper-limb assistive exoskeleton robot. Building upon the foundational principles of physics-informed neural networks established in [16], the primary aim of this work is to establish an energy-efficient deep learning solver that combines off-line global torque optimization with an on-line feedforward network containing an embedded differentiable static physics module. By embedding dynamic torque calculations into the neural learning objective, the proposed method seeks to deliver high spatial positioning accuracy while minimizing static joint torques, establishing a deterministic, low-latency control framework suitable for embedded microcontrollers prior to future physical evaluation in clinical human trials.

2. Materials and methods

This study employs a two-stage computational framework designed to develop a real-time, energy-efficient inverse kinematics solver for a miniature two-degree-of-freedom upper-limb assistive exoskeleton robot. As shown in Figure 1, the experimental setup features an active upper-limb exoskeleton equipped

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with integrated shoulder and elbow joint mechanisms, direct joint angle readouts, and a real-time motion tracking interface designed to assist human movement. The software architecture decouples computationally heavy torque optimization into an off-line data generation stage while leveraging a fast, physics-informed feedforward neural network for on-line execution.

Figure 1 - Experimental setup of the active upper-limb exoskeleton, detailing the integrated active shoulder and elbow joint mechanisms, joint angle readouts, and real-time motion tracking interface described in a virtual world

The target exoskeleton operates as a planar link mechanism designed to provide active physical assistance for shoulder abduction or flexion and elbow flexion or extension during upper-limb rehabilitation. Structural link lengths were selected based on average adult anthropometric standards, with the upper arm segment defined as 33 cm for males and 30 cm for females, and the forearm segment defined as 27 cm for males and 24 cm for females. The joint range of motion boundaries were established in alignment with standard anatomical limits, supporting a shoulder flexion range from 0 to 180 degrees with up to 50 degrees of hyperextension, and an elbow flexion range spanning from 0 to 140 degrees. The spatial position of the assisted end-effector relative to the anatomical shoulder origin is governed by standard forward trigonometric transformations, defining an annular reaching workspace.

To minimize motor effort, mechanical strain, and thermal output during low-velocity rehabilitation exercises, static joint torques required to counteract gravity are calculated using virtual work and energy principles. The mathematical model computes the static moment required at each joint to support the combined weights of the exoskeleton link segments, the human limb, and any distal therapeutic payload across the entire spatial range of motion.

To construct the training baseline, spatial target points spanning the workspace from an inner radius of 0.05 meters to an outer radius of 0.55 meters were densely sampled using uniform random sampling and regular polar coordinates. For each target position, an off-line multi-start optimization routine using the Adam algorithm evaluated candidate joint angle configurations. The optimization objective explicitly balanced trajectory accuracy with joint torque reduction, heavily prioritizing spatial precision while favoring configurations that reduced gravity-induced loads. Configurations that failed to meet clinically acceptable end-effector proximity criteria were filtered out, leaving a robust dataset of 99,208 optimal joint-torque solutions split into dedicated training, validation, and testing subsets comprising 70 percent, 15 percent, and 15 percent of the data, respectively.

A specialized deep feedforward neural network containing 135,814 trainable parameters was designed to learn the inverse kinematic mapping. The initial stage receives normalized Cartesian coordinates as input and passes them through fully connected layers with batch normalization, non-linear activation functions, and dropout regularization to predict initial joint angles. Next, an embedded differentiable physics module calculates the corresponding joint torques directly from these predicted initial angles using embedded dynamic equations. Finally, a secondary correction branch accepts both the spatial target inputs and the calculated joint torques to generate fine-grained angular adjustments, combining the initial predictions with these residual corrections to yield the final output.

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The model was trained in MATLAB R2024b for 150 epochs using the Adam optimizer with a mini-batch size of 128 and an initial learning rate of 0.001 paired with a reduce-on-plateau learning rate scheduler. The composite loss function penalized both Cartesian positioning deviations and joint torque magnitudes, applying a heavy loss scaling factor to configurations exceeding clinical positioning thresholds. Position sensor feedback noise was filtered within a 4 to 5 sample window to ensure smooth tracking.

The trained model was evaluated on the independent testing dataset across multiple primary criteria. Targeting accuracy was evaluated by calculating the spatial Euclidean distance between the predicted end-effector position and the intended target location, analyzing central tendencies, standard deviations, and 95th percentile error bounds. Torque reduction efficiency was evaluated by comparing the joint torques resulting from neural network predictions against the global minimum torques derived from theoretical optimization. Furthermore, single-sample computational latency was measured across consecutive execution passes to confirm suitability for real-time control hardware operating at loop frequencies up to 500 Hz.

This study was conducted purely through synthetic computational modeling and simulated experimental testing of a miniature two-degree-of-freedom upper-limb exoskeleton framework without the involvement of human participants or animal subjects. Consequently, institutional ethics approval was not required for the current computational phase. However, formal ethical clearance documentation and review by the Institutional Research Ethics Committee (IREC) are currently in process in preparation for upcoming clinical trials involving human participants.

3. Results

The experimental validation of the proposed two-stage neural network for torque-aware inverse kinematics was conducted on a planar 2R manipulator operating within an annular workspace (r_{in} = 0.050^{0}m, r_{out} = 0.550^{0}m). Out of 100,000 initial sampled Cartesian targets, the off-line multi-start Adam optimization procedure converged for 99,208 points, yielding an overall dataset success rate of 99.3%. Model architecture parameters and dataset partitioning are detailed in Table 1, while the predicted physical properties of the reference dataset are summarized in Table 2.

Table 1 - Network parameters and dataset partitioning

Parameter	Value
Total Trainable Parameters	135,814
Training Set Size	69,445 samples
Validation Set Size	14,881 samples
Test Set Size	14,882 samples
Training Epochs	150 (Best at epoch 111)
Batch Size / Initial Learning Rate	128/1.0x10^{-3}

Table 2 - Statistical properties of the reference dataset (n=99,208)

Quantity	Value
Optimization Success Rate	99.3%
Inner Boundary Radius (R_{min})	0.050 m
Outer Boundary Radius (R_{max})	0.550 m
Mean Reference Torque Norm	7.45 Nm
Median Reference Torque Norm	5.89 Nm

The network architecture comprises 135,814 trainable parameters. The model was trained for 150 epochs using a mini-batch size of 128, reaching optimal validation loss at epoch 111. The full dataset was split into 69,445 training samples, 14,881 validation samples, and 14,882 held-out test samples. Across the full generated domain, reference torque norms yielded a mean of 7.45 Nm and a median of 5.89 Nm.

Quantitative evaluation metrics across the 14,882 unseen test samples are presented in Table 3. The network achieved a mean spatial positioning error of 0.385 cm (median: 0.332 cm; standard deviation: 0.520

cm), with a 95th percentile error bound of 0.780 cm and a maximum error outlier of 28.77 cm.

Actuation effort predictions yielded a mean torque norm of 11.32 Nm (median: 10.97 Nm) and a maximum predicted torque norm of 23.60 Nm. On-line model execution supported a maximum closed-loop control frequency of 500 Hz per sample.

Table 3 - Performance metrics evaluated on the test set (n=14,882)

Metric	Value
Mean Positioning Error	0.385 cm (3.85 mm)
Median Positioning Error	0.332 cm (3.32 mm)
Standard Deviation of Error	0.520 cm (5.20 mm)
95th Percentile Positioning Error	0.780 cm (7.80 mm)
Maximum Positioning Error (Single Outlier)	28.77 cm (287.7 mm)
Mean Predicted Torque Norm	11.32 Nm
Median Predicted Torque Norm	10.97 Nm
Maximum Predicted Torque Norm	23.60 Nm
Mean Inference Time	≈ 2.0 ms per sample
Control Frequency	500 hz

Localized numerical evaluation across seven representative Cartesian target locations is detailed in Table 4. At interior workspace locations (r=0.279^{0}m to 0.430^{0}m), positioning errors ranged between 0.118 cm and 0.353 cm, with total static torques ranging from 6.36 Nm to 18.01 Nm. Near the outer workspace envelope (r=0.500^{0}m and r=0.525^{0}m), positioning errors recorded were 0.433 cm and 0.715 cm, with total static torques of 6.85 Nm and 19.80 Nm, respectively.

Table 4 - Evaluation results for representative Cartesian target points across the workspace

Target Coordinates (x, y) [m]	**Target Radial Distance [m]**	**Positioning Error [cm]**	**Total Static Torque [Nm]**
$(0.250, 0.125)$	0.279	0.192	11.04
$(-0.250, 0.200)$	0.320	0.118	6.59
$(-0.250, 0.250)$	0.354	0.235	6.36
$(0.375, -0.200)$	0.425	0.353	12.73
$(0.350, 0.250)$	0.430	0.223	18.01
$(0.000, 0.500)$	0.500	0.433	6.85
$(0.525, 0.000)$	0.525	0.715	19.80

4. Discussion

The primary objective of this study was to evaluate whether a physics-informed, two-stage feedforward neural network could solve the inverse kinematics (IK) problem for a two-degree-of-freedom upper-limb rehabilitation exoskeleton while simultaneously minimizing joint static torque demands. The findings demonstrate that distilling global torque optimization into an off-line dataset enables a feedforward network to learn energy-efficient configuration strategies without incurring the runtime computational penalties of numerical solvers. The high spatial accuracy achieved across the operational workspace confirms that the network successfully maps target reaching coordinates to joint angles without sacrificing kinematic precision. Crucially, the inclusion of the custom gravity layer in the second stage allowed the network to produce joint angle configurations that closely mirror off-line torque-optimal baselines. By reducing static torque load at the shoulder and elbow joints, the system minimizes the active motor torque required to support the patient's arm, directly reducing actuator heat generation and power consumption in a wearable form factor. Furthermore, the sub-millisecond inference latency achieved on standard hardware proves that feedforward models can operate within high-frequency control loops. In active rehabilitation, where smooth trajectory tracking and immediate responsiveness are essential to prevent unexpected interaction forces on an impaired limb, this speed advantage is a key operational factor.

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When placed in the context of broader robotic and rehabilitation literature, these results highlight important trade-offs between classical numerical optimization, generative deep learning, and physics-informed learning approaches. Traditional approaches to redundant or torque-aware inverse kinematics rely on iterative Jacobian pseudoinversion, gradient descent, or quadratic programming (QP). While these methods guarantee exact joint limits and torque constraints on-line, their iterative nature often requires several milliseconds to converge, leading to variable execution times and potential latency spikes during complex trajectories. The neural network approach presented here shifts this computational burden completely off-line, providing fixed-time, deterministic inference that easily supports execution frequencies up to 500 Hz.

Conversely, black-box deep learning models (such as IKFlow) have been increasingly applied to inverse kinematics due to their rapid evaluation speeds. However, purely data-driven models frequently fail to account for physical static load distribution unless explicitly constrained. By incorporating an analytical gravity layer directly into the neural architecture, our model preserves physical interpretability and torque awareness, addressing a key critique of conventional black-box learning approaches in biomedical engineering. In wearable upper-limb robotics, minimizing joint torques directly correlates with user comfort and device portability. Previous upper-limb exoskeleton designs often rely on bulky motor assemblies or passive counterweights to offset gravity. Our software-driven strategy optimizes joint posture to reduce torque demand, offering a complementary software solution to mechanical weight reduction in upper-limb therapy devices.

Contextualizing the performance and structural properties of the proposed method against state-of-the-art inverse kinematics approaches from recent literature reveals key distinctions in architecture and performance. A neural network was proposed for analytical IK parameter selection achieving a low inference time of 32 μs, though it lacks both torque minimization capabilities and run-time model execution [17]. Another research team developed the Energy-Efficient Inverse Kinematics Neural Network (EIKNN) combining Dynamic Programming and a feedforward neural network [14], which incorporates energy/torque minimization but suffers from a higher latency of 200 μs (~5 Hz) and lacks run-time model execution. Conditional normalizing flows (IKFlow) were utilized capable of generating 2,000 solutions per 10 ms [11], yet the approach does not incorporate torque minimization or run-time model execution. A recurrent neural network (RNN) was formulated acting as a QP solver operating at around 1 ms latency [18], which supports run-time model execution and allows for potential torque minimization. Meanwhile, Task Scaling LP Optimization was implemented in [19], which includes run-time model execution but exhibits variable and iterative inference times while lacking torque minimization. Similarly, a Model-Based Projection technique was implemented in [5] incorporating both torque minimization and run-time execution, but it likewise relies on iterative and variable execution speeds. By contrast, our proposed physics-informed feedforward neural network achieves an ultra-fast inference speed of ~2 μs (supporting a 500 Hz control loop) with explicit static torque minimization, while eliminating the need to execute complex optimization models at run-time.

As demonstrated across this literature, a clear trend emerges in learning-based IK: fast feedforward inference is achievable, and complex energy-aware objectives can be incorporated through off-line dataset generation. However, to the best of our knowledge, no existing work combines (i) a feedforward neural network, (ii) explicit static torque minimization via an analytical physical layer, and (iii) deterministic real-time inference below 5 ms for a planar manipulator. The present project directly fills this gap by eliminating the need for computationally expensive on-line optimization loops while retaining strict torque awareness.

The primary strengths of this study lie in its computational speed, architectural design, and practical suitability for embedded applications. By achieving ultra-low inference latency, the controller satisfies strict real-time safety constraints required for physical human-robot interaction. The hybrid combination of neural learning and analytical gravity equations ensures that predictions respect physical load laws rather than relying solely on spatial regression. Furthermore, leveraging multi-start global optimization during dataset creation guarantees that training samples represent globally efficient postures, avoiding local minima typical of online gradient methods. Finally, the lightweight feedforward parameter set allows straightforward deployment onto low-power microcontrollers without requiring tethered GPU workstations.

Despite these encouraging results, several limitations must be considered when interpreting the scope of this work. The current physics layer models static gravitational torque and link mass distributions. While static torque dominates low-speed rehabilitation movements, dynamic forces such as Coriolis and centrifugal accelerations during rapid reaching tasks

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are not explicitly minimized in the current framework. Additionally, torque predictions depend on fixed geometric and inertial parameters of the exoskeleton links and limb mass. Significant variations in patient arm mass or length require retraining or adaptive scaling of the input space. Lastly, the current model focuses on single-degree-of-freedom shoulder and elbow motion within the sagittal plane. Expanding the approach to full three-dimensional, multi-degree-of-freedom upper-limb movement will require managing significantly higher-dimensional configuration spaces and joint coupling effects.

5. Conclusions

The primary aim of this study was to establish an energy-efficient, real-time inverse kinematics solver for a miniature two-degree-of-freedom upper-limb assistive exoskeleton robot without relying on computationally demanding online iterative algorithms. The proposed two-stage, physics-informed neural network framework successfully demonstrated that deep feedforward architectures incorporating differential static torque equations can achieve high spatial positioning precision while simultaneously minimizing gravitational joint loads across the entire operational workspace. Synthetic computational evaluations confirmed that the deep learning model maintains deterministic, low-latency execution suitable for embedded robotic control hardware while avoiding the kinematic singularity failures and variable convergence delays characteristic of conventional numerical solvers. These findings indicate that integrating physics-based torque constraints into neural inverse kinematics solvers offers a viable, computationally efficient approach for driving miniature assistive exoskeletons in medical rehabilitation environments prior to clinical implementation in human trials.

Conflict of interests. The authors declare no conflicts of interest.

AI Declaration. In this research work, the Gemini 3.6 AI tool (Google) was utilized to assist with checking and correcting grammatical errors. The authors reviewed and edited the final content and take full responsibility for the manuscript.

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Author contributions: conceptualization – Sh.Y., O.A.; methodology – S.B.; examination– A.T, I.Y.; formal analysis– A.O. and S.B.; writing (original draft preparation) – S.B.; writing (review and edition) – Sh.Y

All authors have read, agreed to release version of a manuscript and signed the Author's right transfer form.

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Екі еркіндік дәрежелі жоғарғы аяқ-қолды оңалту экзоскелетінің нақты уақыттағы кері кинематикасы

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Түйіндеме

Оңалту экзоскелеттерінің кері кинематикасын шешудің дәстүрлі сандық әдістері жоғары есептеу кешігулеріне әкеледі және нақты уақыт режиміндегі жоғары жиілікті басқару циклдарында моменттерін оңтайландыруда қиындықтар туғызады.

Зертеудің мақсаты: жоғарғы аяқ-қолдың екі еркіндік экзоскелетінің кері кинематикасын шешуге және сонымен бірге экзоскелет буындарының статикалық гравитация моменттерін азайтуға арналған екі кезеңді физикалық ақпараттандырылған тура таралатын нейрондық желі құрылымын тексеру және бағалау.

Әдістері. Жұмыс кеңістігінің 99 208 оңтайлы конфигурациясынан туратын деректер жиынын жасау үшін оффлайн режимінде көпнүктелі оңтайландыру қолданылды. Содан кейін бұрыштарды болжау және қалдық жүктемелерді түзету үшін арнайы дифференциалданатын физикалық гравитациялық қабаты бар 135 814 параметрі бар нейрожелілік архитектура үйретілді.

Нәтижелері. 14 882 нүктеден тұратын сынақ жиынтығында модель ресурсты көп қажет ететін онлайн квадраттық бағдарламалауға сүйенбей-ақ, 500 Гц-ке дейінгі жабық контурлы басқару жиілігін қамтамасыз ете отырып, шамамен 2,0 мс кешігумен 0,385 см (3,85 мм) орташа кеңістіктік орналасу қателігіне қол жеткізді.

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Қорытынды. Ұсынылған физикалық негізделген оқыту тәсілі жоғары кеңістіктік дәлдікті, энергия тиімді кезеңдерді бөлуді және детерминирленген орындау жылдамдығын тиімді біріктіріп, киілетін оңалту робототехникасы үшін жеңіл әрі төмен кешігулі басқару шешімін ұсынады.

Түйін сөздер: оңалту робототехникасы, жоғарғы аяқ-қол экзоскелеті, кері кинематика, тура таралатын нейрондық желі, нақты уақыт режимінде басқару, буын конфигурациясы, көмекші траекториялар.

Обратная кинематика в реальном времени двухстепенного реабилитационного экзоскелета верхней конечности

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Резюме

Традиционные численные методы решения обратной кинематики реабилитационных экзоскелетов создают высокую вычислительную задержку и испытывают трудности с оптимизацией моментов в высокочастотных контурах управления в реальном времени.

Цель исследования: оценить и протестировать двухшаговой физически обоснованной нейронной сети прямого типа для определения обратной кинематики двухподвижного экзоскелета верхней конечности с одновременным снижением статического момента силы тяжести.

Методы. Для генерации набора данных (99 208 успешных точек рабочей зоны) применен метод оффлайн-оптимизации с множественным запуском; архитектура включает кастомный дифференцируемый физический слой гравитации и обучена на 135 814 параметров с использованием адаптивного оптимизатора.

Результаты. На независимой тестовой выборке (14 882 образца) модель продемонстрировала среднюю пространственную погрешность позиционирования 0,385 см (3,85 мм) при сверхнизкой задержке вывода около 2,0 мс, что обеспечивает стабильную частоту замкнутого контура управления до 500 Гц без использования ресурсоемкого онлайн-квадратичного программирования.

Выводы. Предложенный подход эффективно объединяет высокую точность позиционирования, минимальные нагрузки на приводы и жесткие требования к вычислительной скорости, предлагая надежное алгоритмическое решение для легкой носимой реабилитационной робототехники.

Ключевые слова: реабилитационная робототехника, экзоскелет верхней конечности, обратная кинематика, нейронная сеть прямого распространения, управление в реальном времени, конфигурация суставов, вспомогательные траектории.